A Battle of Curves

The stylobate curvature in Greek temple architecture

Sokratis Georgiadis - Maria Georgiadou

On 31 March 1833, the Turkish garrison of the Athenian Acropolis, which counted 250 men under the orders of Osman Effendi, surrendered to the Bavarians. This occurred three years after Greece had been established as an independent state, two months after the appointment of Otto of Bavaria as King of Greece, and three months prior to the decision to make Athens the capital of the new kingdom. Over a year later, on 18 August 1834, the German architect Leo von Klenze persuaded the Bavarian Regency to abolish the status of the Acropolis as a military installation, to take the necessary steps for the removal of “the disturbing ruins and the rubbish heap of modern buildings” from the sanctuary, and to reconstruct, as far as the “circumstances of the existing, considerable remains” permitted, of the Parthenon using the existing, considerable remains. Thus, idealist vision was the revival of the ancient monument in all the classical glory it had attained during Periclean times. The fulfillment of this objective implied the complete elimination of all traces that history had left on the Athenian sanctuary since the 5th century B.C. This operation, however, very soon turned out to be a blessing for research into classical architecture. All of a sudden, certain aspects of this architecture, which had slipped the attention of earlier researchers of antiquity – even of those who had until then been among the most successful, such as the British James Stuart and Nicholas Revett –, became visible. One of these aspects were the curvatures of the biggest building on the Acropolis: the Parthenon.

The English architect J. Pennethorne and, almost simultaneously, the Bavarian government, encouraged Joseph Hoffer at the “Greek Research Society” to examine the “all structural lines of the building, which have hitherto been assumed to be straight and level!” were curves. Shortly thereafter, Hoffer published the results of his observations in Ludwig Förster’s Wiener Allgemeine Bauzeitung. Pennethorne did not release his own insights until six years later, though he did so in a publication that was only intended for private distribution. It was however not before The Principles of Athenian Architecture, a monumental book written by the English architect Francis Cranmer Penrose, that the new discovery attracted scholarly attention. The richly illustrated outcome of extensive research and measurements was published in 1851 by the Society of the Dilettanti. Its rather elaborate subtitle – “The most important curves in point of extent are those which form the horizontal lines of the buildings where they occur; such as the edges of the steps, and the lines of the entablature, which are usually understood to be straight level lines, but in the steps of the Parthenon, and some others of the best examples of Greek Doric, are convex curves, lying in vertical planes; the lines of the entablature being also curves nearly parallel to the steps, and in vertical planes.” However, the extraordinary significance of these lines was, in reality, not only grounded in their greater extension, but originated above all from the fact that they appeared to be entirely new around the mid-19th century. For, in contrast to other “refinements”, the existence of which had long been known, such as the upward diminution of the column or the outward curvature of the column shaft, which the Greeks called *entasis*, knowledge about horizontal curvatures and the practice of their construction had faded into oblivion for centuries (Fig. 1). The Roman architectural theoretician, Vitruvius, may have had a dedicated readership since the Renaissance, but it was not intelligible what he meant exactly when he expressly recommended that the level of the stylobate (i.e. the substructure on which the colonnade stands) “must be increased along the middle by the *scamilli impari*; for if it is laid perfectly level, it will look to the eye as though it were hallowed a little.” (Vitruvius 3.4.5) The method (“by the *scamilli impari*”) suggested by Vitruvius for the construction of this addition (*adjectio*) was even then known, since the corresponding illustration mentioned in the text was missing. Perhaps this background also explains the resistance that Penrose’s 1851 publication met. No other than the famous Berlin architect Carl Friedrich GüOTZ had already warned his contemporaries against Penrose’s assertions: “The curvatures of the Parthenon have, since they have become known, gained a wholly undeserved significance and have led to the most paradoxical inferences. Not only amateurs have, through Penrose’s work, been deluded into believing his re-discovered miracle. Practical architects, who were not so familiar with the nature of ancient construction, have equally been deceived.”

Penrose’s conservative opponents refused to accept that the construction of the curvatures was intended by the ancient architects and asserted instead that these deviations were in reality irregularities that could be traced back to inaccurate construction or deformations caused by the setting of the ground or the external application of violence. The resistance of this group of scholars did not last long. In 1912, when the first book exclusively dedicated to the “Greek Re-Discovery”, appeared, hardly anyone seriously doubted anymore that the curvatures were the result of intent rather than a consequence of chance. Its author was the American William Henry Goodyear, director of the Brooklyn Museum at that time.

But once the problem of intention was solved, a series of other questions came to the fore of interest. One of the most important among these concerned the character of the curves. Were they at all the product of mathematical calculations and, if so, what kind of calculation? The question had already occupied the first researchers. Hoffer, for example, was certain with regard to the stylobate of the Athenian Parthenon, that it was a “segment of a circle” (381), the radius of which he estimated for the west side of the building to amount to 1853 pecheis. Penrose, on the other hand, favoured the parabola (p. 30): “It appears not unlikely that the arc of a parabola may have been used for this purpose, a figure with which the builders were well acquainted, and which in the space of a man’s hand would give full-size ordinates, which would serve as offsets from a straight line to form the required circular arc, with the utmost facility – and in so small an arc with perfect accuracy.” Nevertheless, Penrose left some doubt concerning the character of the curve, and did not rule out the possibility of the circle line. On the other hand, he omitted to disclose whence he had acquired such certainty that the “builders (in Periclean time) were well acquainted” with the construction of a parabola.

However, the debate about the mathematical character of the curve reached a climax in the 1930s. In 1934, Gorham Phillips Stevens, an American architect, archaeologist and former Director of the American Academy in Rome (1917-32), published an essay, in which he connected Penrose’s measurements with the curvature theory of Auguste Francois Choisy, a French engineer

5 Carl Bötticher: Bericht über die Untersuchungen auf der Akropolis von Athen im Frühjahr 1862 [im Auftrage des Unterrichtsministers ausgeführt]. Berlin, Ernst & Korn 1863.


8 The metric system was introduced in Greece by the decree of 6 October 1836. Pechys and metre were set equal to each other. Thus Hoffer’s 1853 pecheis equalled 1853 metres. Information about this decree has been provided by Rená Fassea.

9 Gorham Phillips Stevens, Concerning the Curve of the Steps of the Parthenon, AJA 38 (1934) 533-542. Information from Stevens’s Archive at the American School of Classical Studies in Athens has been provided by Natalia Vogelskoff-Brogan.


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and historian of architecture. In his 1909 French translation of Vitruvius, Choisy had equally interpreted the Vitruvian addition (adiection) as a parabola. But he also put forward the hypothesis that the increments of the vertical ordinates of the parabola followed a sequence of odd numbers. As Stevens then came to explain, it is a parabola where the constant $c$ of the general parabola-formula $x = cy^2$ has the value 1. In this specific case, for $y = 0, 1, 2, 3, 4, \ldots$ the $x$ values become 0, 1, 4, 9, 16, $\ldots$ respectively. The specific parabola may be plotted according to Fig. 2, Stevens continued. And he stated:

"If the point generating the curve be considered as starting at the origin 0, its successive drops (or downward steps) are represented by the successive odd numbers 1, 3, 5, 7, 9, etc., while the point is passing through unit distances horizontally."

Stevens repeated Choisy's argument that the steps formed by the ordinates of this curve possibly correspond with Vitruvius' *scamilli* ("scamillus" meaning "bench", "seat" or "step" in Latin), whereas the latter's characterisation of the *scamilli as impares* (Latin for "uneven, unequal" or "dissimilar") could indeed point to the odd numbers of the steps' increments. If this assumption were right, the Vitruvian method for the construction of the curvature "by the *scamilli impares*" would be explained (Fig. 2).

The advantage of the method would be double:

1. It would enable the determination of the curvature in stylobates of any width, for the horizontal coordinates of this type of curve may be increased or decreased in any proportion; the vertical coordinates would remain unchanged. Thus a parabola "drawn by the method of small steps having risers of successive odd numbers, will coincide with the parabolic curve of any stylobate" (536f.).

2. It offered the architect a simple way of laying out his curves in full size upon the vertical faces of the steps of his building.

To determine the curvature of the top step, for example, he would proceed as follows: He would firstly determine (in real size) the maximum rise of the top front and top side step respectively on separate working drawings in which he would have drawn the respective horizontal distances (e.g. the width of the step, distances of columns) on a smaller scale. He would mark on the four angles of the top step the desired extremities of his curves, connect the markings with straight lines on the vertical faces of the step, and erect vertical lines through relevant points (e.g. corresponding to axes of columns etc.) which he would have previously located on them. Finally he would make the heights of these lines equal to the heights of the corresponding lines on his small scale drawings. The tops of the vertical lines would lay on the desired curve. (540f.)

All this would remain without great meaning, had this type of curve maintained the abstract-theoretical status, which it still had in Choisy. But Stevens asserted that the curve described by Choisy corresponded with the horizontal curvatures of the Parthenon according to the measures established by Penrose. However, he omitted to demonstrate this in detail.

Here, the mathematician Constantin Carathéodory spoke up. He was not the first mathematician to have taken a stance on the question of curvatures. Guido Hauck, a professor of descriptive geometry and graphostatics at the Königliche Technische Hochschule in Berlin, attempted to establish in a treatise on perspective (1879) a connection between curvature and physio-psychological perception. With respect to the "mathematical designation" of the curve, Hauck noted that, regardless of Hoffer and

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Penrose’s preferences, both the cosinus line, which he personally favoured, and the hyperbola equally came into question. He added that, thanks to the weakness of the flexion, all designated curves would more or less harmonise with the given measurements (p. 139).

Carathéodory placed exactly this question, that is, the question about the “mathematical law” concerning the curves of the Parthenon stylobate, at the heart of his reflection. In an essay published in the Archaeological Newspaper in 1931, he referred directly to Gorham Stevens’s article that had appeared three years before: “Mr Stevens’s observations and calculations are basically correct, but this does not prevent us from considering as an impossibility the hypothesis that the architect of the Parthenon wanted to build parabolic curves. The concept of conic sections in general and that of the parabola in particular was significantly posterior to the 5th century B.C.”

Carathéodory did not explain how such “impossibility”, i.e., Stevens’s parabola, could be correct. Instead, he amplified the contradiction by explaining that the curves of the Parthenon were circles with very big radii. At the same time, he added that there was “no other method” in constructing these circles in space “than that which Stevens describes by drawing on the passage he quotes from Vitruvius.” As is known, however, Stevens did not describe a circle but a parabola. Carathéodory subsequently presented a method of approximation for the construction of circle-like curves with big radii and unflinchingly provided the proof that the stylobate curves of the north, east and south sides of the Parthenon were circles. He made use of new measurements, which D. Lampadarios, professor of the Technical University of Athens and director of the surveying office at the Greek ministry of transport, had carried out on Nikolaos Balanos’s behalf, who was a long-time restorer of the Acropolis. Carathéodory found out that the radius of the curvature amounted to 1850 metres at the short sides of the temple, a measurement corresponding rather accurately with Joseph Hoffer’s estimation in 1838 (Hoffer, however, was not mentioned by Carathéodory). The segment of circle of the long sides had a radius of 5561 metres.

Despite all the contradictoriness and the fact that Carathéodory appeared conspicuously indifferent to the problem of whether his theory could be realised on the construction site (for example, with the aid of working drawings under scale), his critique of Stevens contained a point that could not be ignored: How did Iktinos and Kallikrates, the Parthenon architects, manage to lend the form of parabolas to the horizontal curvatures of their building, if, after all, the parabola was unknown as a mathematical concept at that time? Stevens faced this question in an answer to Carathéodory: “It was not until the middle of the 4th century B.C. that the mathematician Menaichmos discovered the parabola, ellipse and hyperbola”, but at the same time he argued: “It hardly seems possible, however, that the conic sections, which possess delightful intricacies, sprang fully formed from the brain of any one man, like fully armed Athena from the head of Zeus. Without doubt he codified and amplified such treatises on the conics as had been written before his day.” Anastasios Orlandos, a Greek architect and archaeologist, later attempted to give an answer to the same question. In his monumental book “The Architecture of the Parthenon”, he claimed that the circle, ellipse, parabola, and hyperbola and thus the four conic sections


12 Gorham Phillips Stevens, The Curve of the North Stylobate of the Parthenon, Hesperia 12 (1943), 135-143.

must have been known since the 6th century B.C. because they were, he asserted, frequently used in architecture. Orlandos mentioned as an example the capitals of the columns of the Apollo temple in Corinth from around 540 B.C.\(^\text{14}\) (Fig. 3) Referring to the relevant statements made by Proklos and Eratosthenes, Orlandos attributed to Menaichmos (mid-4th century B.C.) the invention of conic sections and to Apollonius of Perga (20-200 B.C.) the authorship of the first systematic account on this subject. But he immediately added that Menaichmos’ achievement merely consisted in having codified previous knowledge (132, n.3). Stevens and Orlandos’s arguments would hardly be defendable against the charge of speculation. But this was by no means the only problem of the parabola theory.

For the implementation of the proof, he basically employed the method of his 1934 article, this time plotting the curve at a scale of 1:400, “a convenient scale for our purpose”, as he wrote. He concluded that “[t]he curve in the drawing almost perfectly represents a parabola”. The only concession that Stevens was prepared to make to Carathéodory in this article consisted in the allusion that the architects of the Parthenon employed parabolas for the curvatures, possibly without knowing exactly what they were doing or to which mathematical principle this curve was subject: “Whether he [Iktinos, one of the Parthenon architects] understood that his curve was a parabola must, the writer believes, remain a matter of speculation. Perhaps all that he knew was that he could obtain his curve easily and quickly by the scamilli impares method.” (p. 143)

Stevens’s basic claim, however, did not remain undisputed. In 1991, Lothar Haselberger articulated the kernel of the critique by convincingly pointing out the perils of graphic representation with greatly distorted proportions. In order to make curves of such subtlety accessible to closer observation, he wrote, the curve’s

\(^\text{14}\) H. S. Robinson [Hesperia 45, 1976, 217, no 36] gives an even earlier date: 570-560 B.C.
vertical ordinates are shortened only slightly or not at all, whereas horizontally, the curves are shortened very considerably.

"With such distortion, a parabola indeed remains, as Stevens has shown, a parabola; but an ellipse, for example, would then also look like a parabola! Thus, a parabola-like appearance of a curve in graphic representation in no way presupposes that the actual curve has the form of a parabola. A parabola-like representation is thus again a necessary, albeit not a sufficient condition for the (actual) form of a parabola." And Bronner concluded:

"The much debated question whether the curvature in ancient buildings is to be regarded as an arc of a circle or a parabola has thus found a new solution. It is neither. It is in a catenary, which in a curve as slight as this would be indistinguishable from a parabola."

Bronner thus presented a method, which was, beyond all mathematical speculation, exclusively based on experience, provided credible results and solved nearly all problems raised. One could even imagine the scamilli impares (putting aside all speculation concerning odd numbers) as a method to invert the curve. The only problem left was that the proposed method could be verified on a building which had a rather great temporal distance from the Parthenon.

This problem could only be addressed a quarter of a century later. The German architect and archaeologist Dieter Mertens discovered little cross-marks on the external surface of the euthynteria18 of the temple of Segesta, Sicily. They were scratched in regular distances on practically straight lines. They divided each long side of the euthynteria in eighteen parts and each short side in eight. The straight horizontal lines which they designated met the four corners of the temple at the level of the upper edge of the euthynteria. Mertens assumed that the little cross-marks were zero-points of levelling lines, which served to produce the curvature of the euthynteria and consequently also of the steps of the crepis18. Mertens could thus carry out the following operation: He spanned strings around the euthynteria, from one corner to the other, let them sag to the measure of the curvature’s maximum rise, and obtained thus the full-scale ordinates (they corresponded to the distances between cross-marks and sagging string) of a curve which was practically identical with the actual curve of the euthynteria. For him, it was thus proved that "the curvature had been determined with length of 164.47 metres. The building was erected in the 4th century B.C. and renovated before 146 B.C. It spanned the entire southern side of the Agora. What was astonishing about Bronner’s discovery was not so much the unusually mild ascent of the stylobate’s convex bend, but his statements about how the curve had been constructed: By actual experiments on a large scale plan of the Stoa it was discovered that the curvature coincides with that of a string attached at both ends and allowed to sag in the center. The theory is thus put forth, substantiated by these experiments, that the ancient architects obtained the curvature by the simple method of stretching a string between the extremities of the building and permitting the center to sag to the desired amount. By merely inverting the curve thus obtained, a uniform convex curvature would be laid out in the same way. It is possible that in a building of such large size as that of the South Stoa, the curvature was laid out on a reduced scale, and expanded on the building itself." And Bronner concluded:

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16 O. Bronner, Measurements and Refinements of the South Stoa at Corinth, AJA 53 (1949) 146f.


18 Crepis or Crepidoma = the Greek term for the stepped platform of a Greek temple (Dinsmoor, p. 390).
the aid of a sagging string. Finally, Mertens, hereby referring to Stevens, interpreted as *scamilli impares* the unequal distances between the sagging string and the scratched cross-marks. The construction of the temple of Segesta began in the last quarter of the 5th century B.C. It remained unfinished, probably due to the war of Segesta against Selinunt in 416 (Fig. 4). Thanks to Mertens’s observations, the temporal range of the validity of the curvature’s construction with a string was considerably extended towards the past in comparison with, for example, the South Stoa of Corinth and thus immediately approached the time of the Parthenon.

But even the construction with a string did not remain without contestation. In 1979, the German architect and archaeologist Lothar Haselberger discovered at the Younger Apollo Temple in Didyma (Asia Minor) the oldest preserved Greek architectural drawings. They were scratched on a surface of around 200 square metres on the inner faces of the stone walls of the Adyton (the inner or holiest room of a temple). One of them was the construction drawing of the *entasis* curve for the columns of the temple. The length of the column shaft had been shortened to a simple scale, whereas all other measures appeared in real size. The profile of the *entasis* was drawn – after its maximum rise was fixed – as a simple circular arc with a radius of 3.2 metres. The radii of all shaft cross-sections were thus available in real size. In an essay, which Haselberger wrote with the mathematician Hans Seybold in 1991, he expressed the idea that the stylobate curvature could also be obtained by employing the same construction principle by simply turning the work drawing by 90° (185). The curve resulting from this would be, “according to the mathematical-geometrical definition a segment of an ellipse, resulting from a homogeneously stretched segment of a circular arc” (172). Haselberger then compared the Didyma-method with the Segesta string curve. Despite the fact that the construction at Didyma – just as the string construction – did not presuppose “any knowledge of curve types and mathematical calculations”, the Didyma-method was by far more sophisticated than the string method of Segesta both in terms of planning as in terms of drawing. The string method could also “very well be regarded as a more primitive and perhaps older method, whereas the Didyma construction could be considered a more developed and maybe more recent method that served more delicate requirements.” (182). In view of the more recent date of the Didymaion (the erection of the building began shortly after Alexander the Great conducted his campaign in Asia Minor in 344/343 B.C.), this assumption is not implausible. But so long as it cannot be proven, it will hover in vacuo, as it were.

It is thus evident that scholars attempting to answer the question about the stylobate curvature (and one has to really keep in mind that this merely involves only one refinement-category) are confronted with some very critical problems. The greatest difficulty of all theories up to now is however oversimplification. Joseph Hoffer already hinted at the matter when he wrote in 1838: “Since two different curves – namely that of the long and that of the short side – meet at the corner-stone of each side [of the stylobate], this corner-stone would actually have to be a broken surface, because both curves would have cut each other in the diagonal.” (p.80) In other words, Hoffer clarifies that the curvature is not an issue to be tackled in two dimensions, but has consequences on the entire stylobate surface, which – precisely because of the curvature – cannot be plane, but curved. The stylobate would consequently be a vault. And the problem becomes even more complicated if one keeps in mind that the curvatures of the long and short sides respectively differ from each other. The idea of a curved stylobate surface underlies Orlando (Fig. 5) and Coulton’s attempts at three-dimensional representation of the curvature in exaggerated scale. Notwithstanding, scholars treat the question about the curve, its character and its manner of construction at the edge of the stylobate, or of the steps of the crepis, or of the euthynteria, that is, as if it were a two-dimensional problem. Indeed, the spatial impact of the curvature can hardly be determined in a reliable manner through measurements and observations due to the state of preservation of the monuments. On the other hand, one would have to admit that the question about the essence of the curvature can then hardly be answered in a satisfactory manner.

Since, in turn, the ancient sources do not provide sufficient information about the meaning and purpose of the curvature, the intentions pursued by the ancient architects through this rather elaborate practice remain in the dark. Was it functional considerations that led them, or are aesthetic thoughts to be regarded as the cause of the “refinements” (Penrose), or “optical corrections” (Thiersch), or simply the “niceties” of design (Lawrence)? Or is it finally the symbolic meaning of the curvature, still hidden to us, which would be able to determine and explain its shape?  